# Exact Real Numbers Computations applied to Geometry : *To infinity and beyond*

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Being able to compute exactly with real numbers is of paramount importance in computer science, especially in the field of computer graphics for all geometric computations. For that matter, there exist several libraries dealing with exact real numbers computations, some of them being actually formally proven correct [KS11] in proof systems such as Coq [BC04].

A joint project at the Universities of Strasbourg and Poitiers consists in studying how to compute exactly with real numbers using a discrete model of the Continnum. This model is based on the so-called Harthong-Reeb line [Cho10]. This (continuous) line, which is proven to be isomorphic to $\mathbb{R}$ is built on top of a particular sort of integers, possibly infinite : Laugwitz-Schmieden integers (also known as $\Omega$-integers) which are in fact sequences of natural numbers. Constant sequences of natural numbers represent the usual natural numbers, whereas strictly increasing sequences represent special numbers which do not belong to $\mathbb{N}$ and are key items to the construction of this line.

Other libraries have been developped to deal with the issue of exact real numbers. This includes *Creal* a library about constructive reals based on research by Valérie Ménissier-Morain [MM94, MM05] and implemented in `Ocaml` by Jean-Christophe Filliâtre [Fil00].

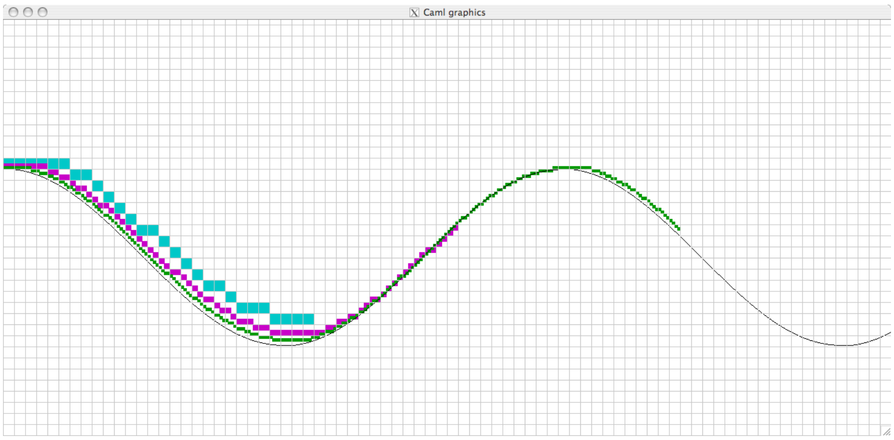


Figure 1 – An example of a real function (cos) and its discrete approximations based on the model of the Harthong-Reeb line.

The goal of this research internship is twofold. On the one hand, it consists in adapting our way of describing exact real computations with the above-mentionned library Creal. To achieve

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that, one shall investigate how to implement all operations available in Creal using our basic operations on reals using Ω-integers and the Harthong-Reeb line. This may include redesigning some parts of our (small) library about Ω-integers.

On the other hand, we want to focus on how to compute *reliably and efficiently* with geometric predicates and their implementation using the Harthong-Reeb line. We suggest to start with the orientation predicate which can be computed using Ω-integers [Yun13] and to study how tools such as filters [MP07] can be used to improve the efficiency of computing such predicates. Indeed, achieving both efficiency and reliability is still a very challenging exercice and usually reliability is achieved at the cost of efficiency. We wish to establish some criteria helping us to finely tune the precision of the computations (namely the rank up to which we actually compute Laugwitz-Schmieden integers) to ensure the results are both reliable and fast to compute. To make sure these computations are correct, we could then formalize these filters on top of the formalization of the Harthong-Reeb line [MCF10] which is developped in the Coq proof assistant.

The internship will take place in the Icube research lab (`http://icube.unistra.fr`) at the University of Strasbourg. The student will belong to the team IGG (*Informatique Géométrique et Graphique*). This research subject could lead to further researches, either in Strasbourg or Poitiers depending on the quality of the applicant and available funding at each site.

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